DYNAMIC ANALYSIS OF A HIGH-RISE BUILDING WITH SEMI-RIGID COLUMN TO BEAM AND BEAM TO COLUMN CONNECTIONS

by

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SYNOPSIS

The purpose of this paper is to study the influence of the connections, beam to column or column to beam, and of the rigidity of those connections on the relative displacement between two adjacent floors and on the maximum story shear in a high rise building subjected to earthquake loading. The moment-rotation characteristics of the column to beam connections are not known but assumed to be identical to the characteristics of the beam to column connections. The results are presented as a comparison between the relative displacement and the maximum story shear in a structure with exclusively beam to column connections, and the relative in displacement and the maximum story shear in a structure with a combination of beam to column and column to beam connections, and the relative displacement and the maximum story shear in a structure with exclusively column to beam connections. The degrees of semi-rigidity in each case are varied to cover a greater spectrum of connections. The influence of axial loads in the columns on the relative displacement is also shown. The above analyses are valid for elastic structures.

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Introduction

The purpose of this paper is to study the influence of the connections, beam to column or column to beam, and of the rigidity of those connections on the relative displacement between two adjacent floors and on the maximum story shear in a high rise building subjected to earthquake loading. Generally steel structures are erected with beam to column connections, thus with continuous columns. Recently structures with continuous beams and continuous columns have been assembled so that intermediary columns are one story high and attached to the continuous beams which are attached to the exterior continuous columns. Three types of framing of the same structure are studied; for each type of framing, four different rigidities of the connections are studied. The influence of the axial load in the columns on the relative displacement and on the story shear is studied.

Moment-Rotation Relations

The equations derived herein are based on the following assumptions: the material is homogeneous, isotropic and linearly elastic; the transverse section of the beam, originally plane, remains plane and normal to the longitudinal fibers of the beam after bending; the deformations are small; the system is conservative.

The beam A-B, shown in Figure 1, is defined by its span, L, its modulus of elasticity, E, and its moment of inertia, I. The loading on this beam is composed of a compressive axial force, P, and of two moments acting at ends A and B of the beam, MA and MB. A lateral load on the beam is described by the bending moment $\bar{M}(x)$, such that $\bar{M}(o)$ and $\bar{M}(L)$ are equal to zero.

The deformation, y(x), of the beam AB is obtained thus:

$$\frac{d^2y}{dx^2} = -\frac{M(x)}{EI} \qquad -1-$$

where M(x) is the bending moment of the beam equal to

$$MA.\left(1 - \frac{x}{L}\right) - MB\frac{x}{L} + P\left(y(x) - f.\frac{x}{L}\right) + \bar{M}(x) \qquad -2-$$

where $f = y(L)$.

The solution of equation -1- with M(x) as in equation -2- is

$$y(x) = C.\cos ax + D.\sin ax - \frac{MA}{P} + \left(f + \frac{MA + MB}{P}\right).\frac{x}{L} - \frac{g(x)}{EI} \qquad -3-$$

where $a^2 = \frac{P}{EI}$

and

$$g(x) = \frac{1}{2ia}\left(e^{iax}\int \bar{M}(x).e^{-iax} - e^{-iax}\int \bar{M}(x)e^{iax}dx\right)$$

with $i = \sqrt{-1}$

Knowing that y(o) is equal to zero and that y(L) is equal to f, the constants are equal to

$$C = \frac{MA}{P}$$

and

$$D = \frac{1}{\sin aL}\left(\frac{g(L)}{EI} - \frac{MB}{P} - \frac{MA.\cos aL}{P}\right)$$

From equation -4- we obtain the moment-rotation relations; the rotation at end A is

$$RA = \frac{MA.E}{K} + \frac{MB.F}{K} + \frac{g(L)}{KL^2}\cdot\frac{u}{\sin u} + \frac{f}{L} \qquad -4-$$

the rotation at end B is

$$RB = \frac{MA.F}{K} + \frac{MB.E}{K} + \frac{1}{K}\left(\frac{g(L)}{L^2}\cdot\frac{u}{\tan u} - \frac{g2}{L}\right) + \frac{f}{L} \qquad -5-$$

where u = aL

$$E = \frac{1 - \frac{u}{\tan u}}{u^2}, \qquad F = \frac{1 - \frac{u}{\sin u}}{u^2},$$

$$K = \frac{EI}{L}, \qquad \text{and} \qquad g2 = \frac{dg(L)}{dx}$$

The values of E and F are tabulated in reference 1. When the values of g(L) and g2 are omitted, $\bar{M}(x) = 0$, in equations -4- and -5-, the resulting rotations are identical to those used by A. Bolton (1,2) and by Livesley and Chandler (3).

Flexible Connections

Experimental studies on the connections between beams and columns in steel structures have shown that the rigidity of those connections varies as a function of the type of connection and of the applied moment as expressed by the moment-rotation curves (4,5,6,7,8,9, and 10). It will be assumed, in this study, that the slope of the moment rotation curve is constant, so that the connections are represented by springs of constant rigidity.

Let RSA and RSB be the rotations of supports A and B, and let KA and KB be rotational rigidities of joints A and B, the end moments are then

$$MA = KA\ (RSA - RA) \quad \text{and} \quad MB = KB\ (RSB - RB).$$

Combining the above equations and equations -4- and -5-, we obtain

$$MA = RSA \cdot SA + RSB \cdot SB - M1 - f \cdot S1 \quad \text{-6-} \quad \text{and}$$

$$MB = RSA \cdot SB + RSB \cdot SC + M2 - f \cdot S2 \quad \text{-7-}$$

where $SA = K \cdot \dfrac{ka \cdot (1 + E \cdot kb)}{DPR}$,

$$SB = -K \cdot \frac{ka \cdot kb \cdot F}{DPR}, \qquad SC = K \cdot \frac{kb \cdot (1 + E \cdot ka)}{DPR},$$

$$S1 = \frac{SA + SB}{L}, \qquad S2 = \frac{SB + SC}{L},$$

$$ka = \frac{KA}{K}, \qquad kb = \frac{KB}{K},$$

$$DPR = 1 + E \cdot (ka + kb) + (E^2 - F^2) \cdot ka \cdot kb,$$

$$M1 = \frac{ka}{DPR} \cdot \left(\frac{g(L)}{L^2} \cdot \left(\frac{u}{\sin u} + kb\ (E-F) \right) + \frac{g2}{L} \cdot F \cdot kb \right), \text{ and}$$

$$M2 = \frac{kb}{DPR} \cdot \left(\frac{g(L)}{L^2} \cdot \left(\frac{-u}{\tan u} + ka\ (E-F-1) \right) + \frac{g2}{L} \cdot (1 + E \cdot ka) \right)$$

As an example the values of g(L) and g2 are for a uniform continuous load of intensity q:

$$g(L) = \frac{q}{a^4}\left(1 - \cos u - \frac{a \cdot L}{2} \cdot \sin u\right)$$

$$g2 = \frac{q}{a^3}\left(\sin u - \frac{u}{2}\ (1 + \cos u)\right)$$

In the case of a tensile axial load, the value a becomes i.a, thus sinu becomes i. sinhu, cosu becomes coshu, and tanu becomes i. tanhu.

The flexibility of the connections are expressed as percentages of a perfectly rigid connection. Let pa be the percentage of rigidity of joint A and pb of joint B, the relation between the percentages and the rigidities is

$$ka = \frac{2pa}{1-pa}, \quad -8- \qquad \text{and} \qquad kb = \frac{2pb}{1-pb} \quad -9-$$

Influence of Axial Loads on Maximum Shear

The influence of axial loads on the static and dynamic behavior of a column has been studied by Lacroix (11). In his work, the author studies the influence of the axial load on the maximum response during forced and free vibrations of a column subjected to a horizontal disturbing force applied at the top end of the column. He concludes that, for free vibrations, the maximum response decreases as the axial load increases. The magnitude of the difference between the maximum response with axial load and without axial load is a function of $\dot{u}_{to}$ which is equal to

$$\frac{Uto}{\dot{U}to \cdot \omega o}$$

where Uto is the initial displacement, $\dot{U}$to is the initial velocity and ωo is the natural circular frequency without the effect of axial load. Figure 2 from reference 11 illustrates the relationship between the ratio of the maximum dynamic response with the effect of the axial load, RDMAX, to the maximum dynamic response without the effect of the axial load, RDOMAX, and the ratio of the axial load to the critical axial load for different values of $\dot{u}$to and for a critical damping of 2% during free vibrations. Figure 3 shows the same relationship for the steady state response of the system where b is the natural circular frequency of the disturbing force.

Types of Beam-Column connections

The structure used to study the influence of semi-rigid column to beam and beam to column connections is shown in Figure 4. Three types of framing have.been studied; they are described in Figures 5, 6 and 7.

The framing of Type 1 is the assembly of five continuous columns and of bay span beams attached through a semi-rigid connection to the columns. This type of framing is the most commonly used in steel construction.

The framing of Type 2 is the assembly of two exterior continuous columns, seven continuous beams attached through a semi-rigid connection to the exterior columns and of story-high columns attached to the continuous beams through semi-rigid connections.

The framing of Type 3 is the assembly of seven continuous beams and of story-high columns attached to the continuous beams through semi-rigid connections.

The assumptions made for each type of framing were that the moment-rotation characteristics of the beam to column and column to beam connections were the same and that the percentage of rigidity was constant for all connections. The percentages of rigidity used were 50, 75 and 90.

Dynamic Analysis

The dynamic response of the structure was obtained by the modal method, in which the responses of the normal modes are determined separately and then superimposed to provide the total response. The stiffness matrix coefficients were obtained from equations -6- and -7- by studying the equilibrium at each joint. The stiffness matrix was then reduced to a matrix of order 7 for the modal analysis. The computer programs needed to obtain the total response are described in references 11 and 12.

In the present analysis, the influence of axial loads was neglected in the computations after evaluating the ratio of the axial load to the critical load. It was found that this ratio was less than 0.03, thus, as can be seen in Figures 2 and 3, the influence of the axial load was minimal. In a taller building however the influence of axial loads could not be neglected.

The earthquake used in the computations was the one recorded at El Centro in 1940, N-S component. Each type of framing was subjected to this earthquake for three different values of the percentages of rigidity of the connections, as stated earlier, to be compared with the results of the analysis of the structure with perfectly rigid connections.

Results

The results presented are the influence of the percentage of rigidity of the connections for each type of framing on the first four natural periods of the structure, on the maximum relative displacement between floors, and on the maximum shear between adjacent stories.

Figures 8, 9, 10 and 11 show the influence of the increasing percentage of rigidity of the connections on the period of the first four normal modes for each type of framing. For the first two normal modes the following observations are true: 1) the period decreases as the percentage increases, 2) the period of Type 1 framing is larger than the period of Type 2 framing which in turn is larger than the period of Type 3 framing. Thus this structure with Type 2 framing is more rigid than with Type 1 framing; the same structure with Type 3 framing would be more rigid than with Type 2 and Type 1 framing. In the third mode the above observations are true when the percentages of rigidity of the connections are equal to 75 and 90; when the percentage of rigidity of the connections is equal to 50, Type 1 framing becomes the more rigid structure and Type 3 framing the more flexible. This later observation is also true in the fourth mode. As could be expected the rigidity of the structure increases as the rigidity of the joints increases. These later findings are also confirmed by Figures 12, 13 and 14, in which are shown the first mode shape for each type of framing with different percentages of rigidity of the connections.

The influence of the percentage of rigidity of the connections for each type of framing on the maximum relative displacement between floors is presented in Tables 1, 2 and 3. The format of those tables is as follows: the first column shows the floor level, the next four columns show the relative displacement for percentages of rigidity of the connections of 100, 90, 75 and 50. The first finding from these tables is that in general the relative displacement for each type of framing increases as the rigidity of the connections decreases. There are however exceptions which result from the nature of the earthquake; remembering that the natural periods vary for each type of framing and for each percentage of rigidity of the connections, the response to a given earthquake is thus different. A second finding is that the relative displacement between upper floors of a structure with semi-rigid connections increases as compared with the relative displacement of a structure with rigid connections. A third finding is the relative displacements of a structure with semi-rigid connections with Type 3 framing are smaller than the relative displacements of a structure with semi-rigid connections with Type 2 and/or with Type 1 framing.

The influence of the percentage of rigidity of the connections for each type of framing on the maximum story shear is presented in Tables 4, 5 and 6. The format of those tables is as follows: the first column shows the floor level, the next four columns show the maximum story shear for percentages of rigidity of the connections of 100, 90, 75 and 50. The first finding from these tables is that, in general, the maximum story shear for each type of framing decreases as the rigidity of the connections decreases. Again there are exceptions due to the nature of the earthquake. The second finding is that the maximum story shear between upper floors of a structure with semi-rigid connections increases as compared with the maximum story shear of a structure with rigid connections. The third finding is that the maximum story-shear of a structure with semi-rigid connections with Type 3 framing is smaller than the maximum story-shear of a structure with semi-rigid connections with Type 1 and for Type 2 framing.

Finally it should be noted that for design purposes, the maximum story shear in the lower part of the structure is obtained when the connections are rigid. However, as can be seen in Table 5, the maximum story shear between floors 5 and 6, 6 and 7 is obtained for connections with a 90% rigidity, this due to the nature of the earthquake. If it is desired to evaluate the possible damage to architectural elements of the building, partition walls, outside walls, window frames and panes, the analysis of the structure with semi-rigid connections would be recommended provided that some data were available on the rigidity of the connections to be used in the structure.

Conclusion

The purpose of this paper was to study the influence of the connections, beam to column or column to beam, and of the rigidity of those connections on the relative displacement between adjacent floors and on the maximum story shear in a high rise building subjected to earthquake loading.

Three types of framing for the same structure were studied: Type 1 framing had exclusively beam to column connections, Type 2 framing had beam to column and column to beam connections, and Type 3 framing had exclusively column to beam connections. For each type of framing different connections rigidities were studied.

The results of the dynamic analysis with the El Centro 1940 N-S earthquake showed that:

1 - A structure with Type 3 framing is more rigid than a structure with Type 2 and Type 1 framing; a structure with Type 2 framing is more rigid than a structure with Type 1 framing.

2 - The rigidity of the structure decreases as the rigidity of connections decreases.

3 - For each type of framing, the relative displacement increases as the rigidity of the connections decreases, and the maximum story shear decreases as the rigidity of the connections decreases.

4 - The relative displacement in a structure with semi-rigid connections with Type 3 framing is smaller than the relative displacement in a structure with semi-rigid connections with Type 2 and/or Type 1 framing; the maximum story shear of a structure with semi-rigid connections with Type 3 framing is smaller than the maximum story shear in a structure with semi-rigid connections with Type 2 and Type 1 framing.

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Floor	Relative displacement (in.)			
0				
	0.799	0.573	0.943	1.144
1				
	1.448	1.116	1.831	2.335
2				
	1.338	1.179	1.721	2.578
3				
	1.171	1.154	1.387	2.368
4				
	1.032	1.123	1.619	2.488
5				
	0.978	1.055	1.752	2.464
6				
	0.726	0.956	1.414	2.261
7				
% of Rigidity	100	90	75	50

Relative displacement of Type 1 framing

Table 1

Floor	Relative displacement (in.)			
0				
	0.799	0.711	0.697	0.720
1				
	1.448	1.384	1.427	1.773
2				
	1.338	1.324	1.372	1.841
3				
	1.171	1.135	1.434	1.422
4				
	1.032	1.143	1.220	1.496
5				
	0.978	1.157	1.110	1.577
6				
	0.726	1.016	1.038	1.319
7				
% of Rigidity	100	90	75	50

Relative displacement of Type 2 framing

Table 2

Floor	Relative displacement (in.)			
0				
	0.799	0.739	0.485	0.471
1				
	1.448	1.441	1.105	1.313
2				
	1.338	1.387	1.222	1.401
3				
	1.171	1.114	1.112	1.425
4				
	1.032	1.114	1.050	1.224
5				
	0.978	1.113	1.136	1.205
6				
	0.726	0.975	0.999	1.118
7				
% of Rigidity	100	90	75	50

Relative displacement of Type 3 framing

Table 3

Floor	Maximum Shear (kips)			
0				
	377.2	262.1	329.5	339.8
1				
	332.2	209.6	285.2	251.1
2				
	259.5	190.7	212.4	204.2
3				
	216.4	178.7	151.3	151.4
4				
	199.9	169.8	186.1	162.9
5				
	176.8	168.7	200.0	167.1
6				
	137.9	158.1	154.1	161.5
7				
% of Rigidity	100	90	75	50

Maximum story shear for Type 1 framing

Table 4

Floor	Maximum Shear (kips)			
0				
	377.2	313.9	290.6	249.6
1				
	332.2	287.4	256.5	235.3
2				
	259.5	234.6	207.9	196.2
3				
	216.4	184.7	201.6	133.3
4				
	199.9	191.6	162.8	147.9
5				
	176.8	195.1	157.1	143.4
6				
	137.9	176.1	146.1	124.2
7				
% of Rigidity	100	90	75	50

Maximum story shear for Type 2 framing

Table 5

Floor	Maximum Shear (kips)			
0				
	377.2	340.9	227.2	206.4
1				
	332.2	305.1	200.8	185.6
2				
	259.5	258.9	203.6	171.7
3				
	216.4	193.2	167.8	159.2
4				
	199.9	194.9	157.6	133.9
5				
	176.8	195.1	164.3	126.8
6				
	137.9	175.7	154.3	122.7
7				
% of Rigidity	100	90	75	50

Maximum story shear for Type 3 framing

Table 6

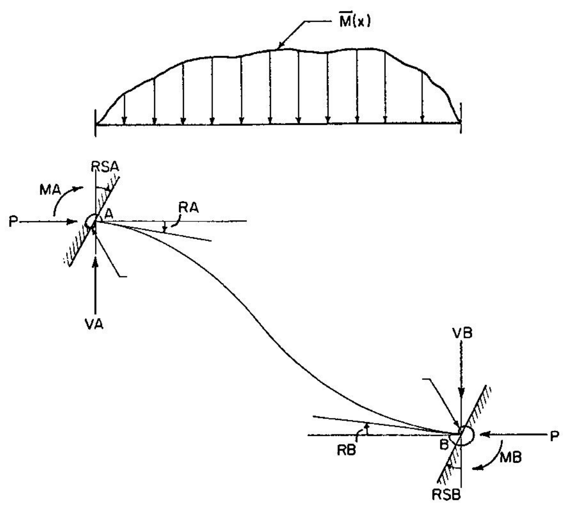

Fig. 1—LOADING ON AND DEFORMATION OF BEAM A-B

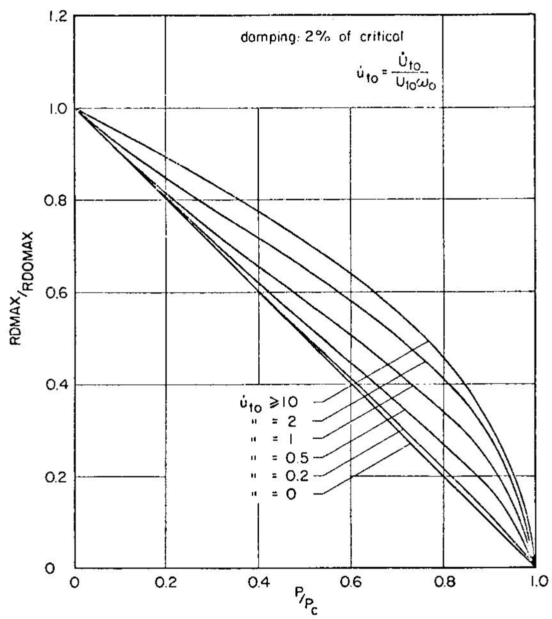

Fig. 2—INFLUENCE OF THE AXIAL LOAD ON THE MAXIMUM RESPONSE DURING FREE VIBRATIONS

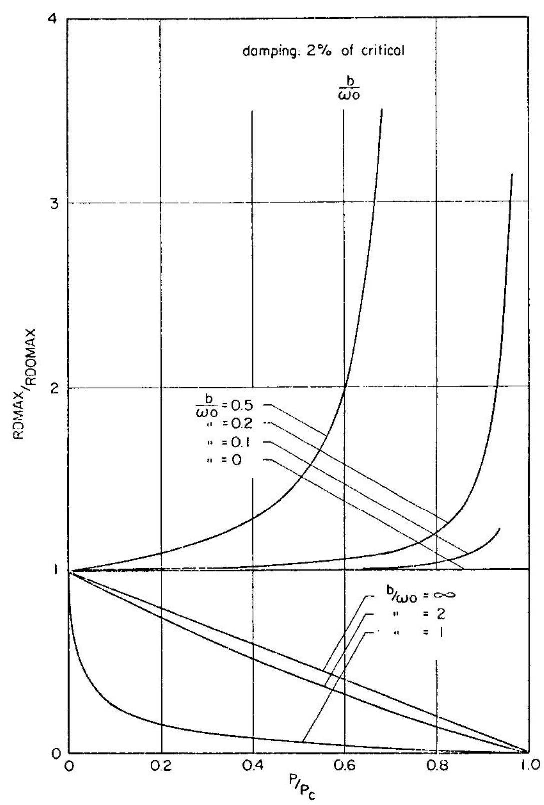

Fig. 3—INFLUENCE OF THE AXIAL LOAD ON THE MAXIMUM RESPONSE DURING FORCED VIBRATIONS

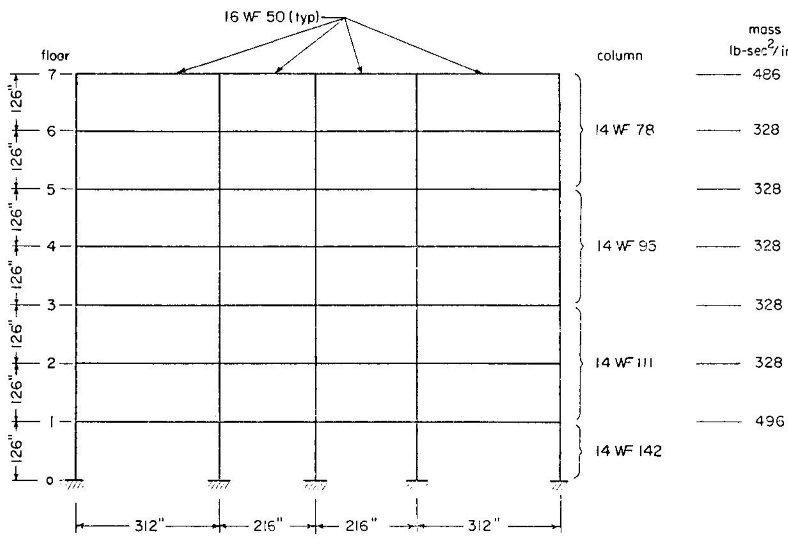

Fig. 4—STRUCTURE TO BE ANALYSED

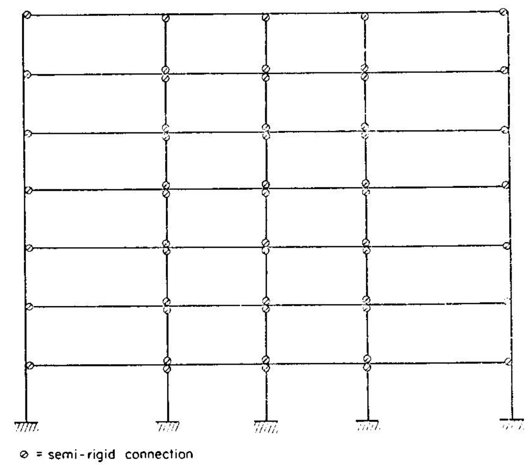

Fig. 6—TYPE 2 FRAMING

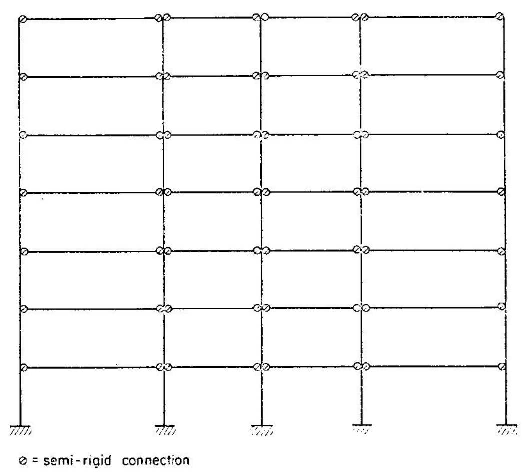

Fig. 5—TYPE 1 FRAMING

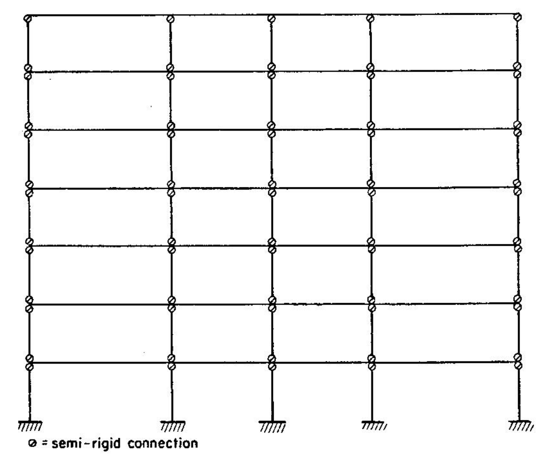

Fig. 7—TYPE 3 FRAMING

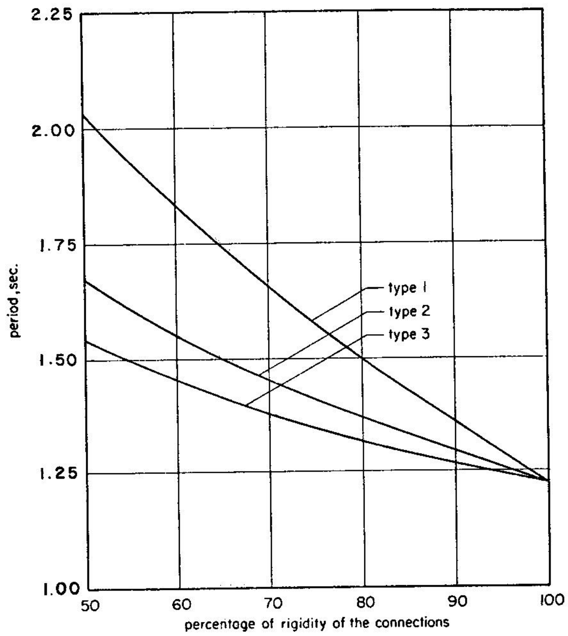

Fig. 8—FIRST MODE PERIODS FOR EACH TYPE OF FRAMING

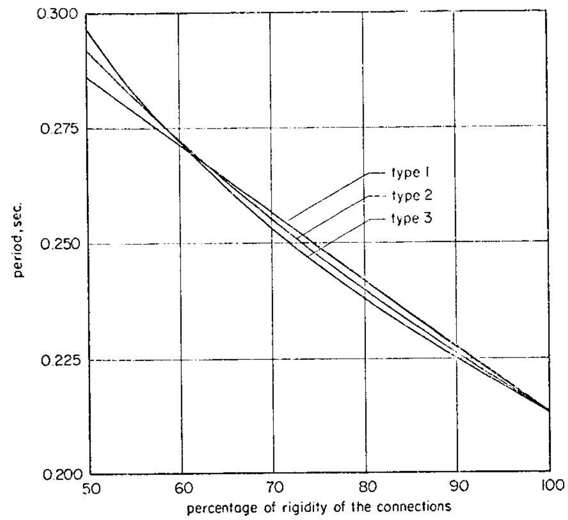

Fig. 10—THIRD MODE PERIODS FOR EACH TYPE OF FRAMING

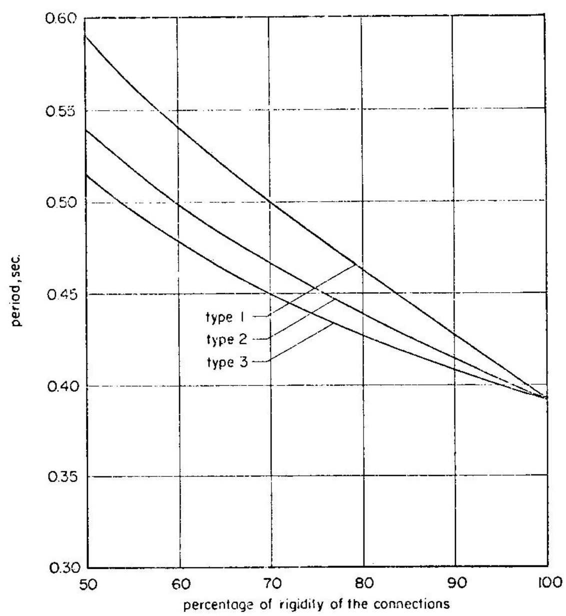

Fig. 9—SECOND MODE PERIODS FOR EACH TYPE OF FRAMING

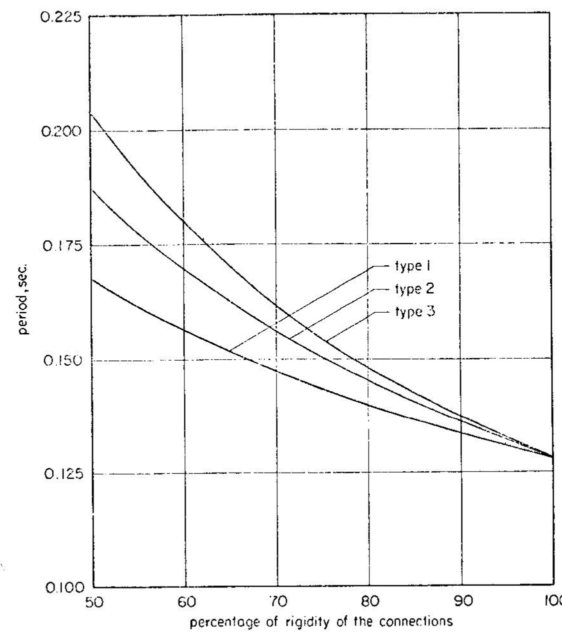

Fig. 11—FOURTH MODE PERIODS FOR EACH TYPE OF FRAMING

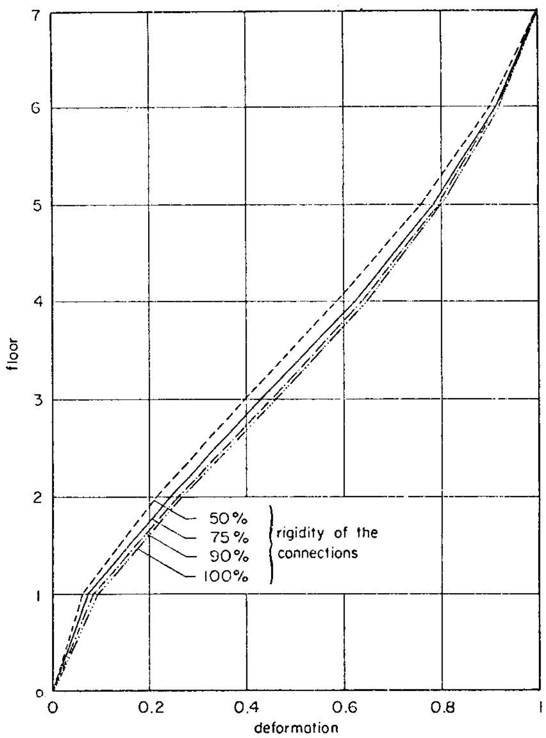

Fig. 12 — FIRST MODE SHAPE WITH TYPE 1 FRAMING

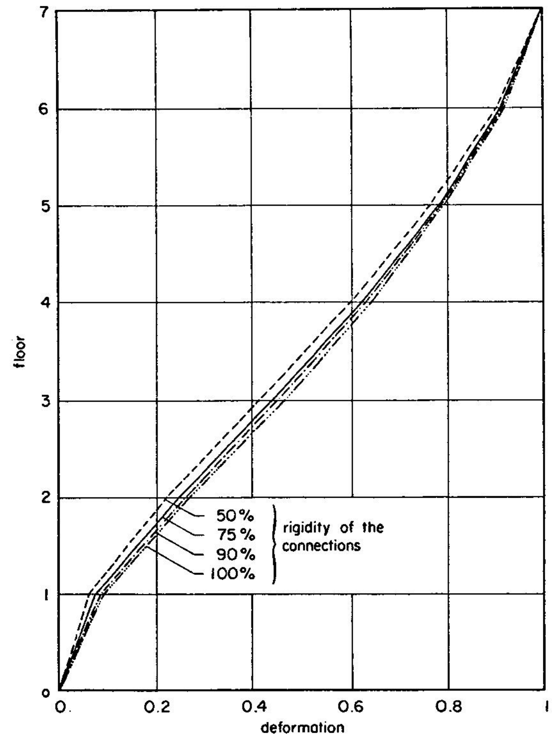

Fig. 13 — FIRST MODE SHAPE WITH TYPE 2 FRAMING

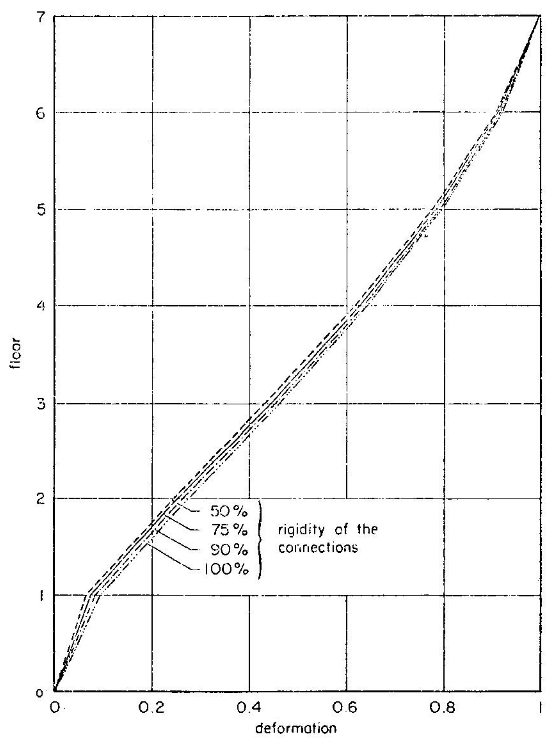

Fig. 14 — FIRST MODE SHAPE WITH TYPE 3 FRAMING